Parallel frequency tracking with built-in performance evaluation

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The problem of estimation of instantaneous frequency of a nonstationary complex sinusoid (cisoid) buried in wideband noise is considered. The proposed approach employs a bank of adaptive notch filters, extended with a nontrivial performance assessment mechanism which automatically chooses the best performing filter in the bank. Additionally, a computationally attractive method of implementing the bank is proposed. The new structure allows one to improve tracking results considerably, especially in nonstationary conditions. In terms of accuracy of frequency estimates, the proposed scheme outperforms existing ones considerably.

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1. Introduction

Tracking of instantaneous frequency of nonstationary narrowband signals is often accomplished using adaptive notch filters (ANFs). A number of algorithms belonging to this class has been proposed in the literature, e.g. [1–9]. Classical designs are based on constrained poles and zeros filters [1] and lattice filters [2]. Tichavský and Händel developed several adaptive algorithms for which they were able to show important theoretical properties, such as statistical efficiency [3,4]. Another, globally stable, estimator was introduced in [10]. Recent contributions include, among others, a complex filter by Regalia. This algorithm has some desirable properties, like unbiased estimation and fast convergence [5]. In [6] the filter developed by Regalia was modified to use different cost function than the original one. This technique allowed the authors to further improve convergence speed of the filter.

In [11] and [12] new adaptation laws were proposed for the classical second order IIR filter. Mean-square error analysis conducted in the papers showed superior performance of the modified algorithms. Another solution, based on Extended Kalman Filter, was proposed in [7]. The state-space model employed by the algorithm covers both frequency and amplitude variation. Furthermore, the filter is equipped with a mechanism to estimate variances of process and measurement noises and therefore can adapt to nonstationary conditions.

Adaptation laws of ANFs typically include the so-called adaptation gains which are usually adjusted – either by the user or by a special algorithm, see e.g. [13] – so as to optimize signal, rather than frequency, tracking performance of the filter. This is a natural and logical choice for many applications, such as removing power interferences from EEG signals [14], tracking harmonic currents [15] or filtering polluted power grid signals [8,9].

However, there exist applications where the instantaneous frequency is the quantity of primary interest. For instance, rotational speed of a combustion engine can be estimated by means of tracking fundamental frequency of acoustic noise or vibration signal generated by the engine. In such a case signal tracking performance of ANF is a secondary issue. Another example of application where the quality of frequency estimates is important is classification of whale whistles which is based on frequency profile of the whistle [16].

Unfortunately the two above mentioned goals (signal and frequency tracking) are conflicting ones – ANFs which exhibits optimal signal tracking performance usually underperform in terms of frequency tracking and vice versa – see e.g. [17] and [18]. Furthermore, it is not trivial to evaluate frequency tracking performance of an ANF during its operation. While the quality of signal estimates can be easily quantified using prediction errors yielded by the filter, such an approach fails in case of frequency tracking. This makes optimization of frequency tracking performance of ANFs a tricky task. The situation becomes even more challenging when the filter is expected to operate in nonstationary conditions, e.g. under time-varying signal to noise ratio. The above difficulties are likely the reason why the problem of automatic optimization of frequency tracking performance has received almost no attention in the literature – existing results are generally limited to analytic evaluations of theoretically (under optimal settings) achievable mean-square errors and experimental comparisons of various approaches.

In the paper it is shown how the frequency tracking performance of an ANF can be evaluated on-line – without any prior
knowledge of the true frequency trajectory. This new result is employed to construct a parallel frequency estimator which consists of a bank of adaptive notch filters and a mechanism for selecting the locally the best performing estimator. Additionally, a computationally attractive extension of the proposed parallel scheme is introduced. The performance of the new solution is evaluated both in absolute and relative terms and is shown to exceed that of existing methods.

The paper is organized as follows: Section 2 presents problem formulation and the ANF algorithm which makes the foundation of the paper. In Section 3 a special predictor which can be used to evaluate frequency tracking performance is constructed. Section 4 proposes a parallel structure, consisting of multiple adaptive notch filters and an appropriate selection mechanism. Extensions are discussed in Section 5. Section 6 presents simulation results. Section 7 concludes.

2. Problem formulation

Consider the problem of estimating the instantaneous frequency of a complex-valued signal

\[ s(t) = a(t)e^{j\int_0^t \omega(t) \, dt} \]  

using noisy measurements

\[ y(t) = s(t) + v(t), \]

where \( t = 0, 1, \ldots \) denotes the dimensionless, discrete time, \( a(t) \) is a slowly time-varying complex "amplitude", \( \omega(t) \) denotes instantaneous frequency and, finally, \( v(t) \) is a wideband measurement noise. Note that, since \( a(0) \) is a complex quantity, it may as well incorporate the initial phase of \( s(t) \). This is the reason behind the absence of initial phase in (1).

The starting point of our discussion is ANF algorithm proposed in [18]. It takes the form

\[ \hat{f}(t) = e^{I(\hat{a}(t-1)+\hat{a}(t-1))} \hat{f}(t-1), \]

\[ \hat{e}(t) = y(t) - \hat{a}(t-1) \hat{f}(t), \]

\[ \hat{a}(t) = \hat{a}(t-1) + \mu \hat{f}^*(t) \hat{e}(t), \]

\[ \hat{\alpha}(t) = \hat{\alpha}(t-1) + \gamma_\alpha \delta(t), \]

\[ \hat{\omega}(t) = \hat{\omega}(t-1) + \hat{\alpha}(t-1) + \gamma_\omega \delta(t), \]

\[ \delta(t) = \text{Im} \left[ \frac{\hat{e}(t)}{\hat{a}(t-1) \hat{f}(t)} \right], \]

\[ \hat{s}(t) = \hat{f}(t) \hat{a}(t), \]  

where * denotes complex conjugation, the quantities \( \hat{a}(t) \), \( \hat{\omega}(t) \), and \( \hat{\omega}(t) \) are the estimates of the signal's complex amplitude, instantaneous frequency and instantaneous frequency rate \( \omega(t) = \omega(t) - \omega(t-1) \), respectively. The parameters \( \mu > 0 \), \( \gamma_\omega > 0 \), \( \gamma_\alpha \ll \gamma_\omega \ll \mu \), are scenario-dependent small adaptation gains determining the rate of amplitude adaptation, frequency adaptation and frequency rate adaptation. Finally, \( \hat{s}(t) \) denotes the filtered estimate of the narrowband signal \( s(t) \).

In spite of its simplicity, the gradient frequency tracking algorithm (3) has very good statistical properties. As shown in [18], under the following assumptions:

- (A1) The instantaneous frequency drifts according to the 2-nd order random walk (also called integrated random walk)

\[ \dot{\omega}(t) = \omega(t-1) + \alpha(t-1), \]

\[ \alpha(t) = \alpha(t-1) + w(t), \]

where \( w(t) \) forms a stationary zero-mean white noise sequence;

- (A2) The sequence \( \{w(t)\} \) is Gaussian distributed, \( w \sim \mathcal{N}(0, \sigma_w^2) \);

- (A3) The sequence \( \{v(t)\} \), independent of \( \{w(t)\} \), is a zero-mean complex Gaussian white noise, \( v \sim \mathcal{CN}(0, \sigma_v^2) \);

- (A4) The magnitude of the narrowband signal is constant, \( |s(t)| \equiv \hat{a}_0 \).

the algorithm (3) can be made statistically efficient, i.e. it can reach so-called Posterior Cramér–Rao Bounds\(^1\) which bounds the mean-squared frequency and frequency rate tracking errors. Although closed-form expressions for the optimal values of gains \( \mu, \gamma_\alpha, \gamma_\omega \) do not exist, it was shown that they depend only on the following 'normalized' measure of signal nonstationarity [18]

\[ \kappa = \frac{\sigma_w^2}{\sigma_v^2}. \]  

In practical situations the above parameter is unknown and possibly time-varying. Therefore, adaptation gains of the filter must be hand-tuned using some cost criteria. Unfortunately, when one is primarily interested in frequency tracking, determining the optimal values of the adaptation gains is difficult because it is unknown how to measure frequency tracking performance.

A hand-on approach could rely on minimization of mean-squared prediction errors \( e(t) \) yielded by the filter. This would actually correspond to the optimization of signal tracking properties of the filter [13,19]. However, the settings which minimize signal tracking errors are different from those which minimize frequency tracking errors [18]. This stems from the fact that there is a fundamental difference between signal and frequency tracking. In order to track the signal, it is necessary for both the amplitude and the phase estimates to be accurate. On the other hand, estimation of instantaneous frequency requires one to keep track of phase changes only.

Fig. 1 shows the steady-state mean-squared signal tracking errors \( \Delta s(t) = \hat{s}(t) - \hat{s}(t) \), frequency tracking errors \( \Delta \omega(t) = \hat{\omega}(t) - \hat{\omega}(t) \) and prediction errors \( e(t) \) for different settings of the filter. To reduce the number of degrees of freedom, the gains \( \mu, \gamma_\omega, \gamma_\alpha \) were set according to the following rule of thumb, suggested in [18],

\[ \gamma_\omega = \mu^2/2, \quad \gamma_\alpha = \mu^3/8. \]  

Instantaneous frequency and amplitude of the nonstationary complex sinusoid \( s(t) \) were governed by

\[ \omega(t) = 0.2 + 0.1 \cos(2\pi t/2000), \]

\[ a(t) = 3 + \sin(2\pi t/500), \]

where \( t \in [0, 5000] \) (the first 1000 samples of the output was discarded to guarantee that steady-state conditions were reached). The variance of the wideband noise was \( \sigma_v^2 = 0.01 \).

It is clear from the results that signal prediction errors are not a good measure of frequency tracking performance and a different quantity must be used for this purpose.

3. Assessment of frequency tracking errors

3.1. Preliminary considerations

Before proceeding further, one should first shed some light onto the tracking properties of the filter (3). Let \( |s(t)|^2 = \hat{a}_0^2 \equiv \text{const} \), \( e(t) = -\text{Im} \{ v(t) s^*(t)/\hat{a}_0^2 \} \). Note that, when (A3) holds, \( e(t) \) is a
In the next section, a one-step predictor \( \hat{u}(t|t-1) \) of the signal \( u(t) \) will be derived. This will enable us to evaluate frequency tracking performance of the ANF algorithm without any prior knowledge of the true frequency trajectory.

**Remark.** Note that, even though \( u(t) \) is not directly accessible, it may be recovered, whenever necessary, by means of inverse filtering the estimates \( \hat{w}(t) \)

\[
u(t) = \frac{1}{Q(q^{-1})} \hat{w}(t).
\]  

**3.2. Construction of the predictor**

Our approach will be based on Wiener principles – under (A1)–(A4), and under the optimal settings of filter gains, the predictor proposed below will be optimal as well. Later on, it will be demonstrated with simulations that it also enables one to measure frequency tracking performance for more realistic profiles of frequency variation and under suboptimal settings.

Let \( A(q^{-1}) = (1 - q^{-1})^2 = 1 - 2q^{-1} + q^{-2} \) and \( C(q^{-1}) = (1 - q^{-1}) \). Then

\[
\omega(t) = \frac{w(t-1)}{A(q^{-1})},
\]

\[
u(t) = \omega(t) + C(q^{-1})e(t)
\]

Furthermore, let

\[
S_{xy}(q^{-1}) = \sum_{\tau=-\infty}^{\infty} E[x(t)y(t-\tau)]q^{-\tau}.
\]

The transfer function of Wiener filter which predicts \( u(t+1) \) from \( U(t) = [u(t), u(t-1), \ldots] \) takes the form

\[
X(q^{-1}) = \sum_{i=0}^{\infty} x_iq^{-i} = \frac{G_P(q^{-1})}{W(q^{-1})},
\]

where \( W(q^{-1}) \) denotes the spectral decomposition of \( S_{uu}(q^{-1}) \), i.e. a causal transfer function such that

\[
W(q^{-1}) W(q) = \frac{\sigma_e^2 + A(q^{-1})A(q)C(q^{-1})C(q)}{A(q^{-1})A(q)}
\]

and

\[
G_P(q^{-1}) = \sum_{i=0}^{\infty} g_i q^{-i}
\]

denotes the causal part of the following transfer function

\[
G(q^{-1}) = \sum_{i=-\infty}^{\infty} g_i q^{-i} = \frac{Q_{uu}(q)}{W(q)}.
\]

It is clear from (15) that \( W(q^{-1}) = N(q^{-1})/A(q^{-1}) \) where

\[
N(q^{-1}) = A(q^{-1}) A(q) C(q^{-1}) C(q) \sigma_e^2.
\]

It follows that

\[
G(q^{-1}) = \frac{q w(q^{-1})}{A(q^{-1})},
\]

\[
G_P(q^{-1}) = \frac{N_P(q^{-1})}{A(q^{-1})},
\]

where \( N_P(q^{-1}) \), \( \deg N_P(q^{-1}) = \deg N(q^{-1}) - 1 \) is the solution of the following Diophantine equation

\[
q a A(q^{-1}) + N_P(q^{-1}) = q N(q^{-1}).
\]
Thus
\[
X(q^{-1}) = \frac{N_P(q^{-1})}{N(q^{-1})},
\]
\[\tag{18}
\]
In Appendix A it is shown that the transfer function \(Y(q^{-1})\) of the optimal, in the mean-squared sense, estimator of \(\omega(t)\) takes the form
\[
Y(q^{-1}) = \frac{F(q^{-1})}{N(q^{-1})}.
\]
\[\tag{19}
\]
Observe that (18) and (19) share the denominator. Furthermore, under (A1)–(A4), for the optimal values of adaptation gains of the algorithm (3) it must hold that \(Q(q^{-1}) = Y(q^{-1})\), i.e.
\[
D(q^{-1}) = cN(q^{-1}),
\]
where \(c\) is some constant. This also means that
\[
X(q^{-1}) = \frac{D_P(q^{-1})}{D(q^{-1})},
\]
\[\tag{20}
\]
where
\[
D_P(q^{-1}) = (2 + d_1 + (d_2 - 1)q^{-1} + d_3q^{-2}
\]
solves
\[
q\beta A(q^{-1}) + D_P(q^{-1}) = qD(q^{-1}).
\]
In order to check how well the performance of the proposed predictor matches the frequency estimation performance of the ANF algorithm (3), two computer simulations were conducted.

In the first simulation, frequency changes were governed by the integrated random-walk model (A1)–(A4) with \(\sigma_0^2 = 10^{-8}\) and \(\sigma_2^2 = 1\), i.e. \(\kappa = 10^{-8}\). A bank of ANFs of the form (3), with adaptation gains optimized for \(\kappa\) ranging from \(10^{-10}\) to \(10^{-4}\) (the optimal values of adaptation gains were found using numerical methods, see [18] for more details) was used for frequency estimation.

Fig. 2 shows the comparison of mean-squared frequency estimation errors yielded by the algorithm (3) for different settings with the corresponding mean-squared prediction errors yielded by the proposed predictor
\[
\xi(t) = u(t) - \hat{u}(t|t - 1).
\]
Note the agreement between shapes of both curves. In particular the minima of both curves coincide.

In the second simulation, the profiles of instantaneous frequency and amplitude were modified to
\[
\omega(t) = 0.2 + 0.1 \cos(2\pi t/2000),
\]
\[
\sigma(t) = 3 + \sin(2\pi t/500).
\]
The gains of the filters were now set as follows: \(\mu_k \in [0.05, 0.25]\), \(\gamma_{0,k} = \mu_k^2/2\), \(\gamma_{0,k} = \mu_k^2/4\). The results of the experiment, shown in Fig. 3, confirm that the proposed predictor can be used to assess the performance of algorithm (3) filters in a realistic setup.

\[\tag{21}
\]

### Table 1
Summary of the proposed parallel filterbank.

| ANF bank |
|---|---|
| For \(k = 1, 2, \ldots, K\) |
| \(f_k(t) = q\delta_{\gamma\omega}(t) + \delta_{\gamma\alpha}(t)\)| |
| \(\xi_k(t) = y(t) - \hat{\xi}_k(t)\)| |
| \(\hat{\xi}_k(t) = \hat{\xi}_k(t - 1) + \mu_k f_k(t)\xi_k(t)\)| |
| \(\delta_k(t) = \lim_{\mu_k \to 0} |\mu_k| f_k(t)\)| |
| \(\hat{\xi}_k(t) = \hat{\xi}_k(t - 1) + \gamma_{0,k} \hat{\xi}_k(t)\)| |
| \(\hat{\xi}_k(t) = \hat{\xi}_k(t - 1) + \delta_k(t) + \gamma_{0,k} \hat{\xi}_k(t)\)| |

Selection of the best filter
\[
\hat{\xi}_k(t) = \frac{1}{\sum_{k=1}^{K} |\xi_k(t)|^2} \xi_k(t)
\]
\[
k(t) = \arg\min_{k=1, 2, \ldots, K} \sum_{i=t-M+1}^{t} |\xi_k(i)|^2
\]
\[
\omega(t) = \omega(t) + \sigma(t)
\]
\[\tag{22}
\]

4. Parallel estimation

Denote by \(T(t) = [t - M - 1, t]\) the local evaluation window consisting of \(M\) samples. Combining (3), (12) and (20) one can propose the parallel frequency tracking scheme summarized in Table 1. The scheme consists two main components: a bank of \(K\) adaptive notch filters of the form (3) with gains \(\mu_k\), \(\gamma_{0,k}\), \(\gamma_{0,k}\) and the following mechanism for selection of the locally the best performing filter
\[
k^*(t) = \arg\min_{k=1, 2, \ldots, K} \sum_{i=t-M+1}^{t} |\xi_k(i)|^2
\]
where \(\hat{\xi}_k(t) = u(t) - \hat{u}_k(t|t - 1)\). The formula used to compute \(\hat{\xi}_k(t)\) stems directly from (12) and (20)
Authorized in Table 2. It consists of three parts: the pilot filter which
poor.

\[ \omega(t) = \frac{1 - 2q^{-1} + q^{-2}}{\gamma_{\omega,k} + (\gamma_{\omega,k} - \gamma_{\omega,k})q^{-1}} \hat{\omega}_k(t), \]

where \( Q_k(q^{-1}) \) and \( X_k(q^{-1}) \), \( k = 1, 2, \ldots, K \), denote the transfer functions (12) and (20) with their coefficients corresponding to the gains \( \mu_k, \gamma_{\omega,k}, \gamma_{\alpha,k} \) of the \( k \)-th ANF in the bank.

5. Extensions

In this section an extension which enables one to reduce the computational cost of the parallel structure considerably is proposed.

Using (9) it is possible to replace a bank of ANFs with a synthetic bank of \( K \) virtual ANFs, governed by

\[ \hat{\omega}_k(t) = Q_k(q^{-1})u(t), \quad k = 1, 2, \ldots, K, \quad (23) \]

where \( u(t) \) is obtained using the ‘pilot’ ANF (3) and \( Q_k(q^{-1}), \quad k = 1, 2, \ldots, K \) denote the transfer functions (10), each with different values \( \mu_k, \gamma_{\omega,k}, \gamma_{\alpha,k} \).

Fig. 4 shows the comparison of mean-squared steady-state frequency tracking errors yielded by the true and virtual ANFs for different settings of adaptation gains and three choices of the pilot filter. Top plot: pilot filter with \( \mu = 0.02 \). Middle plot: pilot filter with \( \mu = 0.03 \). Bottom plot: pilot filter with \( \mu = 0.1 \).

The variance of wideband noise \( \nu(t) \) was equal to 1.

The results, obtained by time-averaging the interval [1001, 5000], show good agreement between performance of real and virtual filters, even under low signal to noise ratio and in the case where tracking performance of the pilot filter was quite poor.

The extended parallel tracker based on virtual filters is summarized in Table 2. It consists of three parts: the pilot filter which provides preliminary frequency estimates, the virtual bank which reprocesses the pilot results and, finally, the selection mechanism introduced earlier.

The proposed extended structure compares favorably in terms of computational costs with a bank of ‘real’ ANFs. A single ANF requires: 16 real-valued multiplications, 13 real additions/subtractions, 2 real divisions and 2 trigonometric \( \sin, \cos \) operations. The virtual bank of \( K \) filters requires 16 multiplications, 13 additions, 2 divisions and 2 trigonometric operations to implement the pilot filter plus additional 11\( K \) multiplications and 9\( K \) additions (assuming that one virtual filter in the bank duplicates the pilot).

Note that the number of multiplications and additions favors the virtual structure for \( K > 2 \). However, the most important factor contributing to improved computational performance is the near-elimination of divisions and expensive trigonometric operations from the scheme. This property can be invaluable in many embedded applications.

6. Computer simulations

6.1. Evaluation of tracking accuracy for integrated random-walk type changes

In the first simulation experiment, the performance of the proposed parallel structure will be evaluated in absolute, rather than relative, terms. This can be done using frequency trajectory governed by the integrated random-walk model (A1)-(A4).

The parallel estimator employing a bank of ANFs with adaptation gains optimized for \( \kappa \) ranging from \( 10^{-10} \) to \( 10^{-4} \) was used to track the frequency of nonstationary sinusoids with different values of \( \kappa \). Performance of the estimator was evaluated for two widths of the local evaluation window: \( M_1 = 150 \) samples and \( M_2 = 300 \) samples, respectively.

The results, obtained using combined time (\( t \in [10000, 20000] \)) and ensemble averaging (100 realizations of \( v(t) \) and \( \nu(t) \)), are summarized in Table 3. The values of Posterior Gramé–Rao Bound are also provided as a reference performance. Note the near-optimality of the parallel estimator, especially for high values of \( \kappa \). Somewhat poorer performance for smaller values of \( \kappa \) can be attributed to the fact that the evaluation window was gradually becoming too short to reliably pick up performance differences between individual filters in the bank.
6.2. Frequency tracking in nonstationary conditions

In the next simulation, performance of the proposed extended scheme was measured using the following two-mode signal

\[
\omega(t) = \begin{cases} 
0.3 & \text{for } t < 2000, \\
0.3 + 0.1 \cos(2\pi t/1000) & \text{for } t \geq 2000, 
\end{cases}
\]

\(a(t) = 3 + \sin(2\pi t/500).\) (24)

Note that the adopted variation model incorporates two sources of nonstationarity: magnitude variation and frequency variation. The variance of wideband measurement noise in the experiment was \(\sigma_{\eta}^2 = 0.01.\)

The pilot ANF operated under the following setup: \(\mu_0 = 0.05, \gamma_{\alpha,0} = \mu_0^2/2, \gamma_{\alpha,0} = \mu_0^3/8.\) The virtual bank, which consisted of 16 filters, adopted the rule of thumb (6) with \(\mu_0 = 0.05 + 0.01(k - 1), \ k = 1, 2, \ldots, 16.\)

Fig. 5 compares mean-squared frequency tracking errors of individual virtual filters and the final estimate for \(M = 50.\) Note that the accuracy of the parallel scheme is better than that of that of any of the virtual filters.

Fig. 6 shows a typical trajectory of \(k_\nu\) during simulation experiment. Observe that, during the initial transient (\(t \approx 300\) period), the faster converging filters are selected. Then the scheme gradually switches to more accurate, but slowly converging filters. Eventually, \(k_\nu\) settles at 1, except a few brief moments (each less than 10 samples). However, after the signal mode changes, the filters in the bank are selected according to how well they ‘match’ the local rate of nonstationarity of the signal.

6.3. Comparison with other solutions

The parallel scheme was compared with complex adaptive notch filter proposed in [5] and its improved version from [6]. The two-mode signal (24) was used for evaluation purposes.

For a fair comparison, the settings of the two competing filters should be optimized. Note that it is unclear what cost criterion should be used for this purpose. Clearly, truly optimal settings can be found by minimizing the mean-squared frequency tracking error. However, such an approach cannot be used in practice – it requires one to know true frequency trajectory (since no results similar to those obtained in this paper exist for both algorithms). A realistic approach involves minimization of cost criteria \(J\) used to derive adaptation laws of the filters. As will be demonstrated, this results in a considerable performance loss.

Optimization was performed by exhaustive search of the filters’ parameters – pole radius \(\rho\) and adaptation gain \(\mu\) – for \(\rho \in [0.8, 0.999], \ \mu \in [0.0001, 0.005]\) and for the two above discussed approaches.

The results are summarized in Table 4. First, observe that both estimators performed worse than the proposed parallel solution, even under optimal settings. Second, note that if the parameters of the filters were adjusted using a realistic approach, the performance loss reached nearly three orders of magnitude. This shows the superiority of the proposed solution.

Finally, a comparison against a self-tuning algorithm from [13] was performed. The algorithm adapts estimation gains of the filter in such a way so as to minimize mean-squared prediction errors yielded by the filter, i.e. so as to optimize signal tracking performance [13,19]. The resulting mean-square frequency tracking error was equal to \(8.20 \cdot 10^{-6}\), which is – again – considerably worse than the result obtained for the proposed approach.

| Algorithm | \(E[|\Delta \omega(t)|^2]\) |
|-----------|-----------------------------|
| Parallel estimator | 7.42 \(10^{-7}\) |
| Complex ANF, MSE-optimal settings | 1.09 \(10^{-6}\) |
| Complex ANF, \(J\)-optimal settings | 4.11 \(10^{-4}\) |
| Modified complex ANF, MSE-optimal settings | 1.10 \(10^{-6}\) |
| Modified complex ANF, \(J\)-optimal settings | 4.20 \(10^{-4}\) |
7. Conclusions

The problem of estimation of time-varying frequency of a complex sinusoid buried in noise was considered. It was argued that adaptive notch filters often underperform in this task because of absence of an appropriate way to measure frequency-tracking performance of the filter. The paper introduced such a measure for a particular adaptive notch filtering algorithm. Furthermore, using this result a parallel structure with capability to select the filter which performs best in terms of frequency tracking was proposed. A computationally attractive extension of the parallel algorithm was introduced as well. Simulation results confirmed that the performance of the new solution considerably exceeds that of existing approaches.

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Appendix A. Proof of (19)

The transfer function of the Wiener estimator of \( \omega(t) \) takes the form

\[
Y(q^{-1}) = \sum_{i=0}^{\infty} W_{\omega,i} q^{-i} = \frac{H_F(q^{-1})}{W(q^{-1})},
\]

where

\[
H_F(q^{-1}) = \sum_{i=0}^{\infty} h_i q^{-i}
\]

denotes the causal part of the following transfer function

\[
G(q^{-1}) = \sum_{i=-\infty}^{\infty} h_i q^{-i} = \frac{S_{\text{sum}}(q)}{W(q)}
\]

Since

\[
S_{\text{sum}}(q) = \frac{\sigma_w^2}{A(q^{-1}) A(q)}
\]

it is clear that

\[
H(q^{-1}) = \frac{\sigma_w^2}{A(q^{-1}) N(q)}.
\]

Therefore

\[
H_F(q^{-1}) = \frac{F(q^{-1})}{A(q^{-1})},
\]

where \( F(q^{-1}) \) is the minimal solution of the following Diophantine equation

\[
A(q^{-1}) F(q^{-1}) + q F(q) N(q) = \sigma_w^2.
\]

As a result

\[
Y(q^{-1}) = \frac{F(q^{-1})}{N(q^{-1})}.
\]

References


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